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Transition to spatiotemporal chaos in convective flows

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Abstract. The transition to spatiotemporal chaos in thermal convection is described and analysed by studying the properties of local and global variables. Beyond the transition point for spacetime chaos the system displays thermodynamics properties in Fourier space. It is shown that a suitable free energy accounts for the experimental results.

1. Introduction

The transition to spatiotemporal chaos in fluid instabilities presents different features depending on the system under study. It is often associated either with defect motion or with long wavelength couplings and it has to be distinguished from fully developed turbulence that, instead, implies an energy cascade into small length scales [1]. In this paper we describe an experiment in which the spacetime evolution of Rayleigh–Benard convection [2] has been studied in order to investigate some general properties of the transition to spatiotemporal chaos in extended systems and to compare them with the mathemathical models [3–7] in which these properties are normally studied.

To illustrate the general features of Rayleigh–Benard convection, let us consider a fluid layer confined between two horizontal solid plates and heated from below. When the temperature difference between the two plates ΔT is smaller than the threshold value ΔT_c, there is no fluid motion and the heat is transported across the layer only by conduction. In contrast when ΔT exceeds ΔT_c a steady convective flow arises, producing a pattern of parallel rolls with a well defined wavenumber q that is of the order of $3.11/d$, where d is the depth of the layer. The roll axes are parallel to the shortest side of the cell containing the fluid.

Increasing ΔT other instabilities that destabilize the main set of rolls may appear and finally the fluid motion becomes time dependent. We are interested in studying these transitions and the evolution toward chaotic and turbulent states of the time-dependent regimes. Other information about Rayleigh–Benard convection may be found in standard text books and review papers [2].

In the experiment that we describe in this paper the horizontal fluid layer has an annular geometry. Indeed with this geometry and a suitable choice of the horizontal sizes of the cell, it is possible to construct a pattern that is almost a one-dimensional chain of radial rolls (roll axes along radial directions, see also figure 1) with periodic boundary conditions. These features of the spatial pattern are very useful in order to compare the results of our experiment with those obtained in one-dimensional mathematical models [3–7]. Specifically the inner and outer diameters of our cell are

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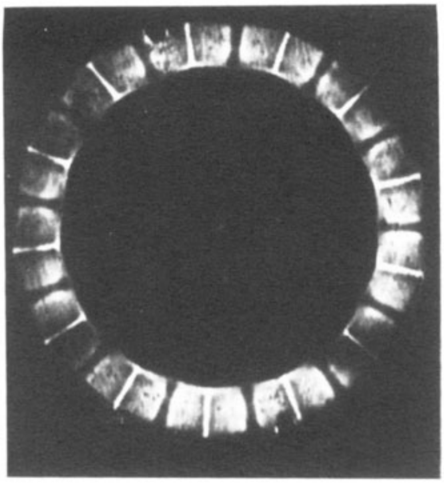
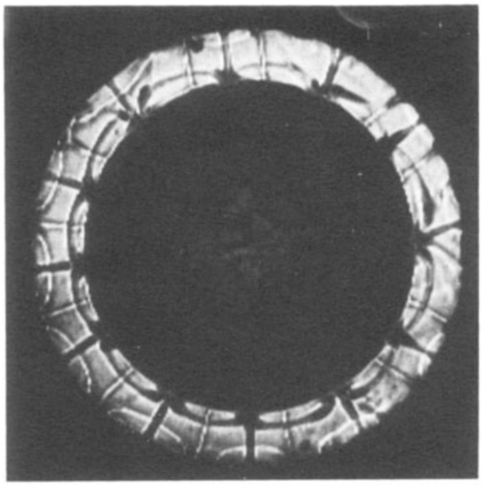


(*a*)



(*b*)

Figure 1. Shadowgraphs of typical spatial patterns. White and dark regions correspond to cold and hot currents respectively. (*a*) Stationary spatial pattern at $\eta = 13$. (*b*) Snapshot of the spatial pattern at $\eta = 230$ in a time-dependent regime (spacetime intermittency).

6 cm and 8 cm respectively, whereas the depth of the layer is 1 cm. The working fluid is silicon oil with a Prandtl number of 30 and the computed value of ΔT_c is 0.06°C.

As an example of the spatial organization of the fluid motion inside our cell, at different values of $\eta = \Delta T/\Delta T_c$, we report in figures 1(*a*) and (*b*) the shadowgraphs of the convective patterns seen from above. Dark regions correspond to the hot currents rising up and white regions to the cold ones, going down. The picture in figure 1(*a*) is the image of a typical stationary structure at $\eta = 13$, i.e. rather close to the convective threshold. Figure 1(*b*) is instead a snapshot of chaotic spacetime evolution, at $\eta = 230$. The shadowgraph technique is very useful to have qualitative information about the spatial structures. To quantitatively characterize the spacetime evolution of the system we measure, on the circle of mean diameter, the two horizontal components of the thermal gradient, averaged along the vertical direction. The measurements have been done by means of another optical method [8]. In what follows we will focus attention only on the component of the gradient perpendicular to the roll axis. This component is called $u(x,t)$, with $0 < x < 1$, where 1 is the whole length of the circle of mean diameter. Other details about the experimental apparatus can be found in [9, 10].

2. Spatial patterns in time-dependent regimes

Analysing the fluid behaviour as a function of $\eta = \Delta T/\Delta T_c$, we observe, that for η around 1, the spatial structure has about 22 rolls. This number increases with η and reaches 38 at η around 200. A detailed analysis of the wavenumber selection process has been reported elsewhere [10].

The spatial structure remains stationary for $\eta < 164$ where a subcritical bifurcation to the time-dependent regime takes place. For $\eta > 164$ the time evolution is chaotic but, reducing η, the system presents either periodic or biperiodic oscillations, and at

$\eta = 149$ it is again stationary. In the range $149 < \eta < 200$ the time dependence consists of rather localized fluctuations that slightly modulate the convective structure, which mantains its periodicity. The fractal dimension and the orthogonal decomposition [11] indicate that the number of degrees of freedom involved in the dynamics is around 3, thus confirming that this dynamic is produced by a small number of degrees of freedom. In these regimes the correlation length is of order one. These measurements indicate that low-dimensional chaos is associated with a spatial order.

At higher η the spatial order begins to be destroyed because of the appearance of bursts detaching from the boundary layer. This regime appears at $\eta = \eta_c = 200$ where also the correlation length begins to decrease. The snapshot, shown in figure 1(*b*), corresponds to such a regime. They present several domains where the spatial periodicity is completely lost (we will refer to them as turbulent) and other regions (that we call laminar) where the spatial coherence is still mantained. This mixture of laminar and turbulent regions is also called spatiotemporal intermittency [4], and clearly appears in our system at $\eta = \eta_s = 248$ [9].

3. Statistical properties of spatiotemporal chaos

In several papers we have reported that, in our system, the transition to spacetime chaos has the properties of a phase transition [7, 9]. This result has been obtained by studying the statistics of the sizes of the laminar regions. Laminar and turbulent regions were distinguished by reducing the spatiotemporal evolution to a binary code in which one stands for turbulent and zero for laminar [9].

In this section we show that this transition can also be characterized by studying the statistical properties of the Fourier mode amplitudes and of some global quantities that may be computed from the spatial Fourier spectra $S(k,t)$ [12]. Indeed the time-averaged spatial Fourier spectra change [9] as a function of the control parameter and they become broadened for $\eta > \eta_s$. As a consequence the Fourier spectra are good candidates to study the transition to spacetime chaos. A similar approach has been recently proposed also by Hohenberg and Shraiman [1], who suggested using the dissipation–fluctuation theorem to define a temperature of the Fourier modes. This kind of description of spatiotemporal chaos has the advantage of dealing with averaged quantities such as the thermodynamic ones of a system near thermal equilibrium.

In order to construct an analogy of the transition to spacetime chaos with the description of a system near thermal equilibrium we need to ask very simple questions. How does the energy fluctuations scale as a function of the integration volume? Have the Fourier modes and local amplitudes Gaussian distributions? Thus we are interested in knowing the statistical properties of the fluctuations $W(x,t) = u(x,t) - \langle u(x,t)\rangle$ ($\langle\ \rangle$ means time average), of their spatial Fourier transform $\tilde{W}(k,t)$, of the energy $E(t)$ and of a suitably defined entropy $S(t)$. The energy is defined in the following way:

$$E(t, N_v) = \sum_{i=0}^{N_v} u^2(x_i, t)$$

with $2 < N_v < N$ where N is the total number of spatial points. The total energy is $E(t) = E(t, N)$. The dependence of the energy on N_v shows how the root mean square (RMS) value, $\Delta E(N_v)$, of the energy fluctuations scales as a function of N_v, i.e. the volume of integration.

The spectral entropy [12] is instead defined in the following way:

$$\sigma(t) = \frac{S(t)}{S_0} = \frac{-1}{S_0} \sum_{k=1}^{N/2} \Phi_k(t) \log(\Phi_k(t))$$

where $\Phi_k(t) = |\tilde{u}(k,t)|^2/E(t)$, $S_0 = \log(N/2)$ is the equipartition value of $S(t)$ and $N/2$ the total number of Fourier modes. The parameter σ is 1 at the equipartition and 0 when only one mode is excited. Thus $\sigma(t)$ is very useful to see whether the system is ordered or disordered. It is important to stress that $E(t)$ and $\sigma(t)$ are not exactly an energy and an entropy but they behave like these two thermodynamic quantities.

Also the distributions $P(W)$, of the local fluctuations W, and $P(\tilde{W})$ of the Fourier mode amplitudes $\tilde{W}_r, \tilde{W}_i$ (i and r denote the real and imaginary parts, respectively), change near the transition point for spatiotemporal chaos η_s.

Specifically we find $P_4(\tilde{W})$ tends to a Gaussian distribution for almost all the modes for $\eta > \eta_s$. We point out that the same transition does not occur in $P_4(W)$ for all the spatial points, indicating that the local dynamics has not, in general, a Gaussian distribution. The fact that the Fourier mode amplitudes have Gaussian distributions whereas the local dynamics does not, has been also reported in [13] and widely discussed in [1, 15]. The reason of this effect is that the small k Fourier modes are coarse-grained variables of the system because they imply an average over many correlation lengths [1, 14].

We now analyse how the energy fluctuations scale as a function of N_v. For $\eta < 200$ the relative fluctuations $\delta E = \Delta E(N_v)/E(N_v)$ do not follow a well defined law as a function of N_v. In contrast, for $\eta > \eta_c$, we find that δE decreases as a function of N_v as a power law N_v^{μ} that extends over the range $2 < N_v < N$. The exponent $\mu(\eta)$ tends asymptotically to $-1/2$. The value of μ indicates that above η_s the spatial points are statistically independent and $E(t)$ behaves, as function of the number of points, as an additive thermodynamic quantity.

All these findings go toward a thermodynamical description of the transition to spatiotemporal chaos in which the Fourier modes may be considered as an ensemble of non-interacting degrees of freedom. An important question is how a 'generalized temperature' of the system may be defined [14]. The main difficulty arises from the fact that the RMS fluctuations of $\tilde{W}$ are not constant as a function of k but present a high-frequency cut-off [9]. This phenomenon, which occurs in the chaotic behaviour of the Kuramoto–Shivanshinsky equation, makes the definition of the temperature a very difficult and still unsolved problem, because it is not clear what can be done with the modes whose fluctuations decrease exponentially as a function of k [9]. An approach in this direction has been made by Zalesky [16].

Instead, Hohenberg and Shraiman [1] suggested using the dissipation–fluctuation theorem to define the temperature of the system. This implies the knowledge of the linear response function of the system, which may be measured by perturbing it with a very small signal. This approach is certainly very interesting, and several tests have been done in our experiment, but no relevant result has been obtained. Indeed, it is not simple to extract a very small signal (the response to the perturbation) from the natural fluctuations of the system. Even in the case where this is possible, the errors in the calculation of the response function may be very large.

So we propose here an approach that is rather similar to the one of [1], but it uses the natural fluctuations of the system. We know [17] that, for a thermodynamic system at constant pressure and volume, the RMS fluctuations of energy and entropy are proportional to $K_B^{1/2}C_v^{1/2}T$ and $K_B^{1/2}C_p^{1/2}$ respectively, where C_v and C_p are the specific heats at constant volume and constant pressure, T is the temperature of the system and K_B is the Boltzmann constant.

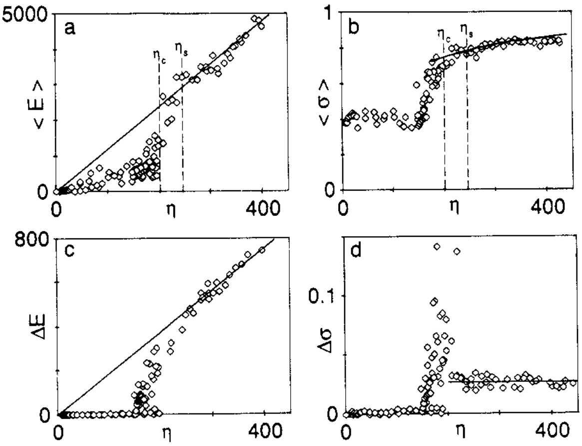


Figure 2. Dependence on η of the mean values of (a) E and (b) σ, and of the RMS values of their fluctuations (c) ΔE and (d) $\Delta\sigma$.

To check these points we report in figure 2 the mean values of E, σ and the RMS values ΔE, $\Delta\sigma$ of their fluctuations as functions of η. We see that $\langle E\rangle$, figure 2(a), and $\langle\sigma\rangle$, figure 2(b), are monotonically increasing as a function of η. The behaviour of σ, above $\eta_c = 200$, indicates that the power spectrum shape does not change as a function of η. From figure 2(d) we immediately realise that $\Delta\sigma$ increases by a considerable amount near η_c. In figure 2(d) we also notice that $\Delta\sigma$ is almost constant above η_s, as a consequence we can make the hypothesis that C_p of our 'thermodynamic system' is constant above η_s. As we cannot distinguish, in our system, between a constant volume and a constant pressure process, we assume $C = C_p \simeq C_v$. Such an hypothesis has to be verified *a posteriori*. In figure 2(c), we see that, for $\eta > \eta_s$, ΔE grows linearly as a function of η (full line in figure 2(c)). As a consequence the ratio $(\Delta E/\Delta\sigma)$ may be considered proportional to the 'generalized temperature' ($\tilde{T} = r\eta$) of the system for $\eta > \eta_s$ where the Fourier mode amplitudes have a Gaussian distribution. From the data we obtain $r = 73 \pm 1$.

In order to demonstrate that our definitions are self-consistent we construct, for $\eta > \eta_s = 248$, a free energy F:

$$F = -Cr\eta\ln(r\eta) + (\sigma_0 + C)r\eta$$

where $\sigma_0 = 0.817 \pm 0.005$, $C = 0.165 \pm 0.005$. From this free energy we may compute $\langle E\rangle$, $\langle\sigma\rangle$ and C as functions of η, via appropriate thermodynamic relationships [16]. The full lines, shown in figures 2(a) and (b), are the result of the calculations, and are in agreement with the experimental points. This verifies the hypothesis used to define the 'generalized temperature' of the system.

The equivalent of the Boltzmann constant may be also computed using $\Delta\sigma$ and C. The result is the following: $K_B = (\Delta\sigma)^2/C = (4.4 \pm 0.2) \times 10^{-3}$.

4. Conclusion

The transition from low-dimensional chaos to weak turbulence has been investigated in Rayleigh–Benard convection in an annular geometry.

In several other papers we have shown that the onset of spatiotemporal intermittency, in our cell, displays features of a phase transition that is reminiscent of a percolation. This result has been obtained by reducing the spacetime dynamics to a binary code that catches the relevant features of the phenomenon. A cellular automaton model, whose transition probabilities have been obtained from the experiment, confirms the exsistence of a phase transition with the exponents of the percolation [18].

The transition to spacetime chaos is accompanied by the appearance of Gaussian distribution for the Fourier mode amplitudes and by a relevant changes of the distibutions of global quantities such the energy and the entropy. In addition, the energy fluctuations decrease when the integration volume increases. A 'generalized temperature' has been defined by using the energy and entropy fluctuations. The existence of a free energy shows the self consistency of our definitions. An open problem is now to understand the meaning of a transition between a regime that displays thermodynamic properties (spacetime chaos) and another that does not have these features.

Acknowledgments

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